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314. Proposed by R. D. CARMICHAEL, Anniston, Ala.

Sum to infinity the series $\frac{1}{2.3.3.4} + \frac{1}{4.5.5.6} + \frac{1}{6.7.7.8} + \frac{1}{8.9.9.10} + \dots$

I. Solution by E. B. ESCOTT, Ann Arbor, Mich., and G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

The general term of the series is $\frac{1}{2n(2n+1)^2(2n+2)}$, which may be resolved into the partial fractions,

$$\frac{1}{4} \left(\frac{1}{n} - \frac{1}{n+1} \right) - \frac{1}{(2n+1)^2}.$$

Therefore,

$$\frac{1}{2.3.3.5} = \frac{1}{4} \left(1 - \frac{1}{2} \right) - \frac{1}{3^2}$$

$$\frac{1}{4.5.5.6} = \frac{1}{4} \left(\frac{1}{2} - \frac{1}{3} \right) - \frac{1}{5^2}$$

$$\frac{1}{6.7.7.8} = \frac{1}{4} \left(\frac{1}{3} - \frac{1}{4} \right) - \frac{1}{7^2}$$

.

Adding, we have

$$\frac{1}{2.3.3.4} + \frac{1}{4.5.5.6} + \dots = \frac{1}{4} - \left(\frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots \right) = \frac{1}{4} - \left(\frac{\pi^2}{8} - 1 \right).$$

[See E. Pascal, *Repertorium der hoeheren Mathematik*, Vol. 1, p. 60.]

The sum is, therefore, $\frac{5}{4} - \pi^2/8 = .0163$.

II. Solution by S. A. COREY, Hiteman, Iowa.

Take the Fourier's cosine series for $x \sin x$, viz.,

$$x \sin x = 1 - \frac{\cos x}{2} - \frac{2 \cos 2x}{1.3} + \frac{2 \cos 3x}{2.4} - \frac{2 \cos 4x}{3.5} + \dots$$

and integrate both members of the equation three times to obtain the only constant of integration involved, viz., $(\frac{1}{2} + \pi^2/12)$, in the series obtained by the second integration. The latter series may then be thus written,

$$-\frac{x \sin x}{2} - \frac{5 \cos x}{4} - \frac{x^2}{4} + \frac{1}{2} + \frac{\pi^2}{12} = \frac{\cos 2x}{1.2.2.3} - \frac{\cos 3x}{2.3.3.4} + \frac{\cos 4x}{3.4.4.5} - \dots (1).$$

When $x=0$, (1) becomes

$$\frac{1}{1.2.2.3} - \frac{1}{2.3.3.4} + \frac{1}{3.4.4.5} - \frac{1}{4.5.5.6} + \dots = \frac{\pi^2}{12} - \frac{3}{4} \dots (3).$$

When $x=\pi$, (1) becomes

$$\frac{1}{1.2.2.3} + \frac{1}{2.3.3.4} + \frac{1}{3.4.4.5} + \frac{1}{4.5.5.6} + \dots = \frac{7}{4} - \frac{\pi^2}{6} \dots (4).$$

Subtracting (3) from (4), we find the sum of the given series to be $\frac{5}{4} - \pi^2/8$.

Also solved by J. Scheffer.

315. Proposed by PROFESSOR B. F. YANNEY, Mount Union College, Alliance, Ohio.

Simplify, $1 - (2 - (3 - \dots - (n-1) - n) \dots))$.

Solution by GEORGE W. HARTWELL, University of Kansas, Lawrence, Kansas, and V. M. SPUNAR, Pittsburg, Pa.

Removing the parentheses, this expression becomes

$$1 - 2 + 3 - 4 + \dots (-1)^{n-1}n \equiv \sum_1^n (-1)^{n-1}n.$$

But $\sum_1^n (-1)^{n-1}n = -(n/2)$ when n is even,

and $\sum_1^n (-1)^{n-1}n = (n+1)/2$ when n is odd.

Also solved by G. B. M. Zerr.

GEOMETRY.

342. Proposed by G. I. HOPKINS, M. A., Instructor in Mathematics and Astronomy, Manchester, N. H.

Given, circle DEF inscribed in triangle ABC and circumscribing the triangle DEF , D, E, F being the points of contact; AH is drawn through center, N , meeting chord DF in H . Through H is drawn BK meeting AC in K . Prove triangle ABK isosceles.

I. Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Let the points D, F, E be situated on the sides a, b, c , respectively, and also let $l = \cos^2(A/2)$, $m = \cos^2(B/2)$, $n = \cos^2(C/2)$. Then $(0, rn, rm)$; $(rn, 0, rl)$, are the trilinear coordinates of D and F , respectively.

Hence $\beta - \gamma = 0$ is the equation to AN , $l\alpha + m\beta - n\gamma = 0$ is the equation to DF .